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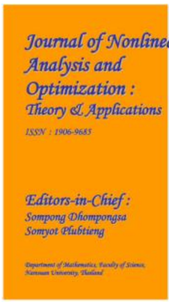

MODULAR SQUARE DIFFERENCE EDGE CORDIAL LABELING OF SOME GRAPHS

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Abstract
A new concept of labeling called the modular square difference edge cordial labeling is introduced and investigated for the lotus inside the circle , the graph [ ], the comb graph ʘ , the crown graph K1,n

Keywords: square difference labeling, square difference edge cordial labeling and modular square difference edge cordial labeling.

1 Introduction

All graphs considered in this paper are finite, simple and undirected graphs. The symbol and denotes the vertex set and edge set of a graph G. The cardinality of the vertex set is called the order of , denoted by p and the cardinality of the edge set is called the size of the graph , denoted by . A graph with vertices and edges is called a graph. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions The concept of graph labeling was introduced by Rosa[14] in 1967. A dynamic survey on graph labeling is regularly updated by Gallian[4]. For standard terminology and notation, we follow Harary[8]. The concept of square difference labeling was first introduced by J. Shiama[15]. Cahit[3] introduced the cordial label of graphs. The concept of difference cordial labeling was first introduced by R.Ponraj[12], A. Alfred Leo[1] introduced the concept of divided square difference cordial labeling graphs. R.Revathi and R.Rajeswari[13] introduced the modular multiplicative divisor labeling of some path related graphs. Motivated by this paper [13], we have introduced a concept of new labeling which modular square difference edge cordial is labeling. A graph is said to have modular square difference edge cordial labeling if there is a bijection and the induced map
{0,1} is defined by if is odd, otherwise 0 and | (0) − (1)| ≤ 1 where ef (0) =edges with label zero,ef (1) =edges with label one Such that n divides the sum of all edge labels of G. If a graph admits modular square difference edge cordial labeling then it is said to be square difference edge cordial graph.

2 Preliminaries

In this section, some basic definitions namely square difference labeling, binary vertex labeling, cordial labeling, square difference edge cordial labeling, path graph Pn,complete bipartical graph K1,n ,the graph [ ], the comb graph ʘ , the subdivision graph S(G), the crown graph and the lotus inside the circle .

Definition 2.1: Let be a graph. A mapping is called the binary vertex labeling of and is called the label of the vertex of under the induced edge labeling is given by for all
Definition 2.2: Let be a graph and e a binary vertex labeling of . The map is called a cordial labeling if | (0) − (1)| ≤ 1 and | (0)
− (1)| ≤ 1 . A graph is called Cordial graph if it admits cordial labeling.

Definition 2.3: Let be a graph. A graph G is said to be square difference

labeling if there exist a bijection such that the induced
function given by for every are all distinct. Any graph which admits square difference labeling is called square difference graph.

Definition 2.4: A graph is said to have square difference edge cordial labeling if there is a bijection and the induced map is defined by if | is odd, otherwise and | (0) − (1)| ≤ 1 where ef(0)=edges with label zero, ef(1)=edges with label one. If graph admits square difference edge cordial labeling then it is said to be square difference edge cordial graph.

Definition 2.5: if the edges and the vertices are distinct in a walk the w is called path. A path on vertices is denoted by .

Definition 2.6: A closed walk in which and are distinct is called a cycle of length .

Definition 2.7: A complete bipartite graph is called a star and it has n+1 vertices and n edges. It is also denoted by .

Definition 2.8: [:] is a graph obtained from a path by joining each vertex of a path to a root of a star S2 by an edge.

Definition 2.9: The lotus inside a circle is a graph obtained from the circle : and a star with central vertex and the end vertices by
By joining each to and ui+1(mod n).

Definition 2.10: The comb is the graph obtained from a path Pn by attaching a pendant edge to each vertex of the path. It is denoted by
Definition 2.11: A subdivision graph S(G) is obtained from G by subdividing each edge of G with a vertex.
Definition 2.12: The crown graph is obtained from Cn by attaching a pendent vertex from each vertex of the graph.

3 MAIN RESULTS
Definition 3.1: A graph is said to admit modular square difference edge cordial labeling if there exist a bijection the induced map is defined by if | is odd, otherwise and | (0) − (1)| ≤ 1 where ef(0)= edges with label zero, ef(1)= edges with label one such that n divides the sum of all edges labels of G. If a graph admits square difference edge cordial labeling then it is said to be modular square difference edge cordial graph.

3.1 MODULAR SQUARE DIFFERENCE EDGE CORDIAL LABELING OF SOME GRAPHS
Theorem 3.1: The comb graph is a modular square difference edge cordial graph.
Proof. Let G be the comb graph

Let } and

{ uivi: }

Hence | V(G)| =2n and |E(G)|=2n+1

Define V(G)→{1,2,3,………2n} as follows.

The induced edge labeling are

1, 1

we have | (0) − (1)| ≤ 1 and n divides the sum of all edge labels of G.
Hence the comb graph is a modular square difference edge cordial graph

Example 3.1: The comb graph is a modular square difference edge cordial graphwhich is shown in the figure 3.1.

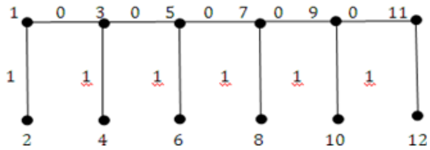

Figure 3.1: The Comb Graph

Theorem 3.2: The graph is a modular square difference edge cordial graph.

Proof. Let G be the graph

Let V(G) ={ u_i, v_i, v_i', v_i'': $1 \leq i \leq n$ } and

E(G) = { u_iu_{i+1} :$1 \leq i \leq n-1$ } U { u_iv_i :$1 \leq i \leq n$ } U { v_iv_i' :$1 \leq i \leq n$} U { v_iv_i'': $1 \leq i \leq n$ }

Hence |V(G)|=$4n$ and |E(G)| $= 4n - 1$.

Define $f: V(G) \rightarrow \{1,2,3,\ldots,4n\}$as follows.

$f(u_i)) = 2i - 1, \quad 1 \leq i \leq n$

$f(v_i)) = 2i, \quad 1 \leq i \leq n$

$f(v_i')) = (2n - 1) + 2i, \quad 1 \leq i \leq n$

$f(v_i'') = 2n + 2i, \quad 1 \leq i \leq n$

The induced edge labeling are

$f^*(u_iu_{i+1}) = 0, \quad 1 \leq i \leq n - 1$

$f^*(u_iv_i) = 1, \quad 1 \leq i \leq n$

$f^*(v_i, v_i') = 1, \quad 1 \leq i \leq n;$

$f^*(v_i, v_i'') = 0, \quad 1 \leq i \leq n;$

We have | (0) − (1)| ≤ 1 and n divides then sum of all edge labels of G. Hence the graph [ ] is a modular square difference edge cordial graph.

Example 3.2: The graph [is a square difference edge cordial graph which is shown in the figure 3.2.

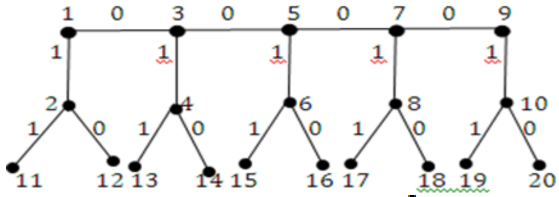

Figure 3.2: The graph [$P_5 : S_2$]

Theorem 3.3: The Lotus inside a circle LC n is a modular square difference edge cordial graph.

Proof. Let G be the Lotus inside a circle .

Let $V(G) = \{v_0, u_i, v_i : 1 \leq i \leq n\}$ and

$$E(G) = \{u_n u_1, u_n v_1\} \cup \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{v_0 v_i : 1 \leq i \leq n\} \cup \{u_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_i : 1 \leq i \leq n\}$$

Hence $|V(G)| = 2n + 1$ and $|E(G)| = 4n$

Define $f: V(G) \rightarrow \{1, 2, 3, \ldots\ldots 2n+1\}$ as follows,

$f(v_0) = 1$

$f((v_i) = 2i + 1, \quad 1 \leq i \leq n$

$f(u_i) = 2i, \quad 1 \leq i \leq n$

The induced edge labeling are

$f^*(u_n u_1) = 0$

$f^*(u_n v_1) = 1$

$f^*(u_i u_{i+1}) = 0, \quad 1 \leq i \leq n-1$

$f^*(v_0 v_i) = 0, \quad 1 \leq i \leq n$

$f^*(u_i v_{i+1}) = 1, \quad 1 \leq i \leq n-1$

$f^*(u_i vi) = 1, \quad 1 \leq i \leq n$

We have (0) − (1)| ≤ 1 and n divides the sum of all edge labels of G.

Hence the Lotus inside a circle is a modular square difference edge cordial graph Example 3.3: The Lotus inside a circle is a modular square difference edge cordial graph which is shown in the figure 3.3.

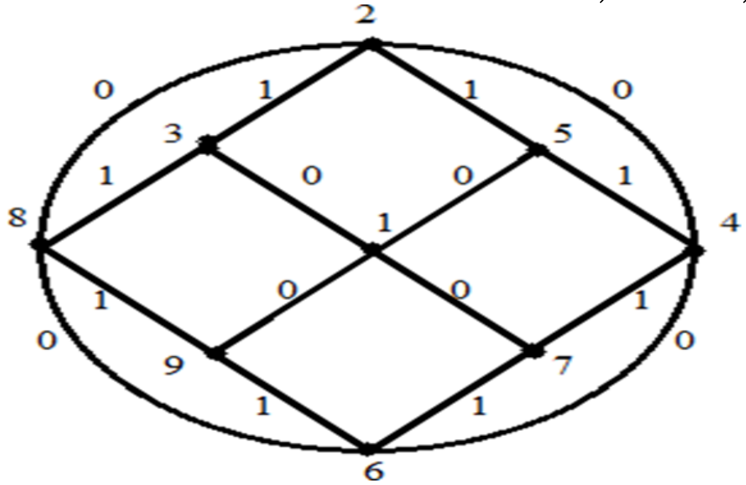

Figure 3.3:

Theorem 3.4: The subdivision of the edges of the star is a modular square difference edge cordial graph.

Proof. Let G be a graph obtained by the subdivision of the edges of the star . Let V(G) = { : } and E(G) ={ : }

Hence |V(G)| = 2n+1 and |E (G)| =2n

Define $f: V(G) \rightarrow \{1, 2, 3, \dots, 2n + 1\}$ as follows

$f(v) = 1$

$f(u_i) = 2i + 1, \quad 1 \leq i \leq n$

$f(w_i) = 2i, \quad 1 \leq i \leq n$

The induced edge labeling are

$f^*(vu_i) = 0, \quad 1 \leq i \leq n$

$f^*(u_iw_i) = 1, \quad 1 \leq i \leq n$

We have $|e_f(0) - e_f(1)| \leq 1$ and n divides the sum of all edge labels of G. Hence the subdivision of the edges of the star is a modular square difference edge cordial graph.

Example 3.4: The subdivision of the edges of the star is a modular square difference edge cordial graph which is shown in the figure 3.4.

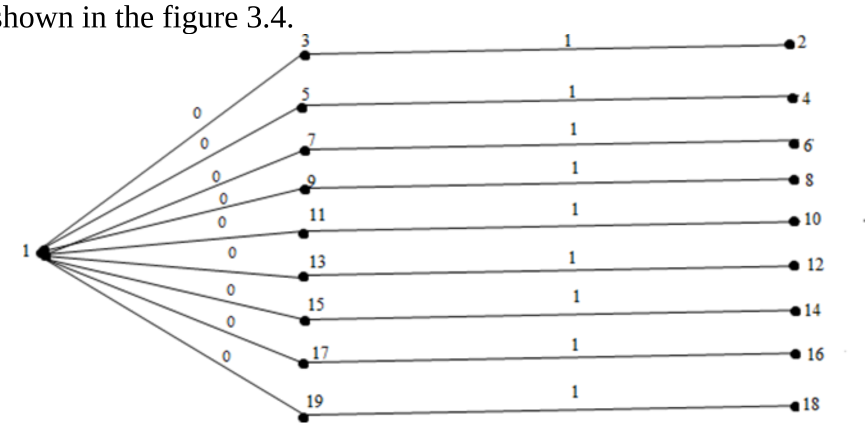

Figure 3.4: subdivision of

Theorem 3.5: The crown graph is a modular square difference edge cordial graph.

Proof. Let G be **the crown graph c_n^+**

Let $V(G) = \{u_i, v_i ; 1 \leq i \leq n\}$ and

$E(G) = \{u_n u_1\} \cup \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_i : 1 \leq i \leq n\}$

Hence |V(G)| =2n and |E(G)| = 2n.

Define $f: V(G) \rightarrow \{1, 2, 3, \ldots, 2n\}$ as follows

$f(u_i) = 2i - 1, 1 \leq i \leq n$

$f(v_i) = 2i, \quad 1 \leq i \leq n$

The induced edge labeling are

$f^*(u_n u_1) = 0$

$f^*(u_i u_{i+1}) = 0, \quad 1 \leq i \leq n-1$

$f^*(u_i v_i) = 1, \quad 1 \leq i \leq n$

We have (0) − (1)| ≤ 1 and n divides the sum of all edge labels of G. Hence the crown graph is a modular square difference edge cordial graph.

Example 3.5: The crown graph is a modular square difference edge cordial graph which is shown in the figure 3.5.

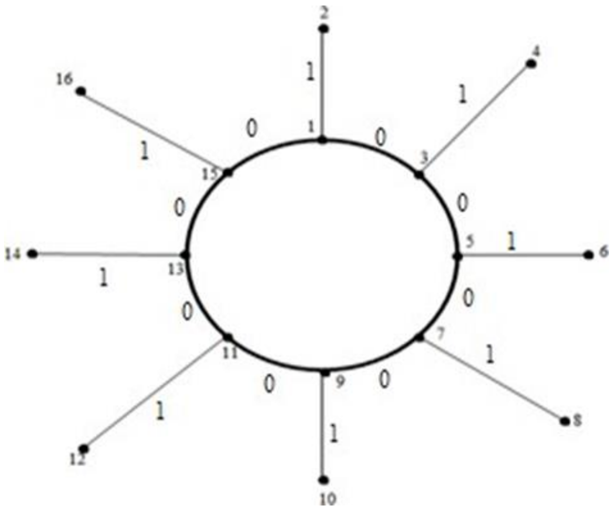

Figure 3.5: The crown C8+

Conclusion

In this paper, the modular square difference edge cordial labeling are introduced and proved for the lotus inside the circle LCn, the graph [ ], the comb graph ʘ ,the crown graph , the subdivision of the edges of the star are modular square difference edge cordial graph. In future, we can proceed with some more types of labeling for different types of graph.

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